OCR
Oxford Cambridge and RSA

Friday 16 June 2023 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

1 **(a)** Express 205 in the form $7q+r$ for positive integers q and r, with $0 \leqslant r < 7$. **[1]**

(b) Given that $7 \mid (205 \times 8666)$, use the result of part **(a)** to justify that $7 \mid 8666$. **[2]**

2 For all positive integers n, the terms of the sequence $\{u_n\}$ are given by the formula

$u_n = 3n^2 + 3n + 7 \pmod{10}$.

(a) Show that $u_{n+5} = u_n$ for all positive integers n. **[2]**

(b) Hence describe the behaviour of the sequence, justifying your answer. **[2]**

3 A surface has equation $z = x^2y^2 - 3xy + 2x + y$ for all real values of x and y.

Determine the coordinates of all stationary points of this surface. **[6]**

4 The equation of line l can be written in either of the following vector forms.

- $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\lambda \in \mathbb{R}$
- $(\mathbf{r} - \mathbf{c}) \times \mathbf{d} = \mathbf{0}$

(a) Write down **two** equations involving the vectors **a**, **b**, **c** and **d**, giving reasons for your answers. **[4]**

(b) Determine the value of $\mathbf{a} \cdot (\mathbf{c} \times \mathbf{d})$. **[3]**

5 **(a)** Express as a decimal (base-10) number the base-23 number 7119_{23}. **[2]**

(b) Solve the linear congruence $7n + 11 \equiv 9 \pmod{23}$. **[3]**

(c) Let $N = 10a + b$ and $M = a + 7b$, where a and b are integers and $0 \leqslant b \leqslant 9$.

(i) By considering $3N - 7M$, prove that $23 \mid N$ if and only if $23 \mid M$. **[4]**

(ii) Use a procedure based on this result to show that $N = 711\,965$ is a multiple of 23. **[2]**

6 When 10^6 of a certain type of bacteria are detected in a blood sample of an infected animal, a course of treatment is started. The long-term aim of the treatment is to reduce the number of bacteria in such a sample to under 10 000. At this level the animal's immune system can fight the infection for itself. Once treatment has started, if the number of bacteria in a sample is 10 000 or more, then treatment either continues or restarts.

The model suggested to predict the progress of the course of treatment is based on the recurrence system $P_{n+1} = \frac{2P_n}{n+1} + \frac{n}{P_n}$ for $n \geqslant 0$, with $P_0 = 1000$, where P_n denotes the number of bacteria (**in thousands**) present in the animal's body n days after the treatment was started.

The table below shows the values of P_n, for certain chosen values of n. Each value has been given correct to 2 decimal places (where appropriate).

n	0	1	2	3	4	5	6	7	8	9
P_n	1000	2000	2000	1333.33	666.67	266.67		25.47	6.64	2.68

n	10	20	40	60	80	100	200	300	400
P_n	3.89	4.67	6.45	7.84	9.03	10.08	14.20	17.36	20.04

(a) Find the value of P_6 correct to **2** decimal places. **[2]**

(b) Using the given values for P_0 to P_9, and assuming that the model is valid,

(i) describe the effects of this course of treatment during the first 9 days, **[1]**

(ii) state the number of days after treatment is started when the animal's own immune system is expected to be able to fight the infection for itself. **[1]**

(c) (i) Using information from the above table, suggest a function f such that, for $n > 10$, f(n) is a suitable approximation for P_n. **[1]**

(ii) Use your suggested function to estimate the number of days after treatment is started when the animal may once again require medical intervention in order to help fight off this bacterial infection. **[1]**

(iii) Using information from the above table and the recurrence relation, verify or correct the estimate which you found in part **(c)(ii)**. **[2]**

(d) One criticism of the system $P_{n+1} = \frac{2P_n}{n+1} + \frac{n}{P_n}$, with $P_0 = 1000$, is that it gives non-integer values of P_n.

Suggest a modification that would correct this issue. **[1]**

7 The group G, of order 12, consists of the set $\{1, 2, 4, 5, 8, 10, 13, 16, 17, 19, 20, x\}$ under the operation of multiplication modulo 21. The identity of G is the element 1. The element x is an integer, $0 < x < 21$, distinct from the other elements in the set.

An incomplete copy of the Cayley table for G is shown below:

G	1	2	4	5	8	10	13	16	17	19	20	x
1	1	2	4	5	8	10	13	16	17	19	20	
2	2	4	8	10	16	20	5	x	13	17	19	
4	4	8	16	20	x	19	10	1	5	13	17	
5	5	10	20	4	19	8	2	17	1	x	16	
8	8	16	x	19	1	17	20	2	10	5	13	
10	10	20	19	8	17	16	4	13	2	1	x	
13	13	5	10	2	20	4	1	19	x	16	8	
16	16	x	1	17	2	13	19	4	20	10	5	
17	17	13	5	1	10	2	x	20	16	8	4	
19	19	17	13	x	5	1	16	10	8	4	2	
20	20	19	17	16	13	x	8	5	4	2	1	
x												

(a) State, with justification, the value of x. **[2]**

(b) In the table given in the Printed Answer Booklet, list the order of each of the non-identity elements of G. **[3]**

(c) **(i)** Write down all the subgroups of G of order 3. **[1]**

(ii) Write down all the subgroups of G of order 6. **[3]**

(d) Determine all the subgroups of G of order 4, and prove that there are no other subgroups of order 4. **[4]**

(e) State, with a reason, whether G is a cyclic group. **[1]**

8 A surface, C, is given by the equation $z = f(x,y)$ for all real values of x and y. You are given that C has the following properties.

- The surface is continuous for all x and y.
- The contour $z = -1$ is a single point on the z-axis.
- For $-1 < a < 1$, the contour $z = a$ is a pair of circles with different radiuses but each having the same centre $(0, 0, a)$.
- The contour $z = 1$ consists of the circle, centre $(0, 0, 1)$ and radius 1.

Sketch a possible section of C corresponding to $y = 0$. **[6]**

END OF QUESTION PAPER

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